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A MAGAZINE DEVOTED TO THE INTERESTS OF TEACHERS OF MATHEMATICS

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CONTENTS

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THE MATHEMATICS TEACHER,
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Professor in the Massachusetts Institute of Technology

and WALTER W. HART
Assistant Professor of Mathematics, Teachers' Training Course, University of Wisconsin

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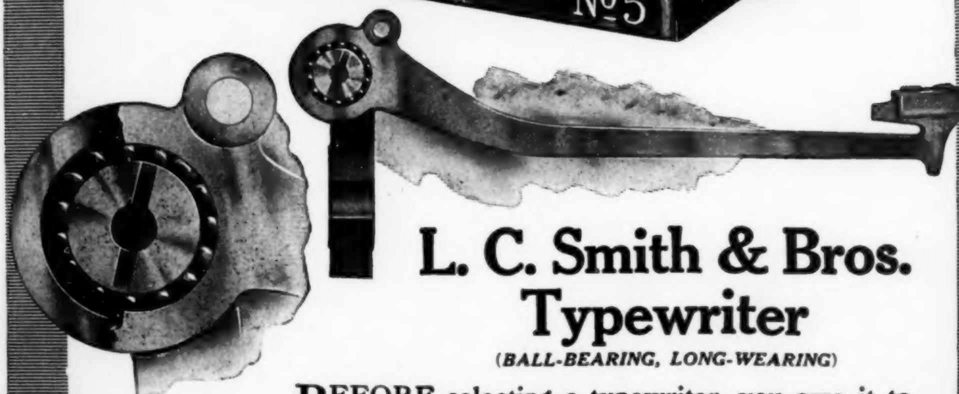

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THE MATHEMATICS TEACHER

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EDUCATIONAL VALUES IN MATHEMATICAL TEACHING.

By H. E. HAWKES.

(Continued from last issue.)

means to an end. It is the lever with which the windows of the mind are pried open, and we teachers must be content to see it discarded when this all-important work is done.

In the present discussion I am not thinking of the needs of the student who will use mathematics professionally. The requirements of the engineer and of the mathematician are comparatively simple and easy to satisfy. It is the general student, who, perhaps, has no idea of even going to college whom I have in mind. The kind of boy who will never use any fact of algebra or geometry more than once or twice in his life, and is then so surprised that he seeks out his old teacher, and tells him all about it—just to make him feel that he has been of some use in the world.

Many of the live questions of mathematical teaching take on new meaning when looked at from the point of view of transferability. I have time to mention but a single one, the problem of the correlation of the various mathematics subjects.

I am certain that all of us have had the chilling experience of asking a question of an algebra class which involves a little geometry. One boy spoke for many when he said: "I am certain that I could answer that question if I were only reciting in

geometry." This merely means that the boy cannot transfer easily even from one class room in mathematics to another in the same department. That is, students are very far from having their complete mathematical knowledge available all of the time. We wish they might. Theoretically it seems very reasonable to suppose that if the hard and fast divisions into distinct subjects were removed, this difficulty of transference would not exist. For it is certain that there is sluggish flow of ideas not only between the subjects of algebra and geometry, but from instructor to instructor, and from class room to class room. So far as this one principle of transferability is concerned, its weight seems to be entirely on the side of a closer correlation of the different mathematical subjects; for such correlation would remove some of the barriers over which transference is difficult.

At many points pertinent and natural relations between algebra and geometry should be emphasized. Even if the boy is in his first year in algebra certain facts from geometry and physics may be stated, and the algebra of these facts worked out with telling effect. And in geometry use should be made of algebra on every reasonable occasion. The fundamental facts of trigonometry may very well be introduced in connection with the work on similar triangles in geometry, and many bridges between analytical geometry and advanced algebra should be constructed. All of this tends to make the line of division between the subjects less sharp and formidable.

But it does not mean that algebra and geometry should be completely amalgamated, or fused, as some of the exponents of the correlation movement call it—confused I should say. The principle of transferability is not the only one which comes into this discussion. The type of imagination, of reasoning, of mental asmosphere which is effective in geometry is quite different from that required in algebra, and I believe that both subjects would suffer from this fusion.

I will not go into details regarding the elements in the second category, that of mental processes, though there is ample material for discussion. But I must remark in passing that the educational ideal of our fathers which they tried to express under the nebulous name of "discipline" really includes much that

the present analysis places in the second and the third category. Compare for educational content this ideal with that of some of us moderns who make the fact, the vocational fact, the ideal of our effort. In spite of its chill and formal sound, the older ideal is vastly deeper and broader and fuller than the modern one.

During the remaining moments let me emphasize some of the moral qualities which mathematics may be expected to evoke, and which are of transcendent educational importance, not only on account of their mathematical value, but on account of their intimate relation to the broad intellectual life of every serious man.

I will mention but three: The respect for truth, the use of the intuition, the appreciation of unity and harmony.

First, the respect for truth. In our subject we draw our conclusions from definitions and postulates which we lay down as absolute, not relative, and the theorems follow from these principles with a certainty which leaves no room for difference of opinion in regard to their validity. The first man to prove a certain theorem has not created it; he has discovered an eternal truth. The fact from which he has drawn the veil has not changed its character, it has merely appeared. There was at the beginning of time a body of mathematical truth, dependent on no man's opinion or intelligence, not pragmatic, but absolute, awaiting the coming of a mind which could lay it bare. I say it should be an impressive experience to the young mind when it comes in contact with a new and unimagined truth, and realizes that from everlasting to everlasting that truth must remain the same. Our formulation of the physical laws is modified as we gain new instruments to discover our past inaccuracies, social ideals are purely relative to the time and place in which we may be, but here is something on which we can depend in the midst of the change or decay of our relative knowledge. This experience is of moral value.

The use of the intuition. In the teaching of mathematics it has always been my custom to urge the student to push ahead of his actual demonstration with the imagination. Daily in explaining a new theorem I ask, "How *ought* it to turn out?" In this way the imagination and the intuition are stimulated, and the reasoning faculties come limping along in time, with the

demonstration. This, it seems to me, is an important matter. Of course students must use their observation and reasoning, but a sure intuition to break the way for the mind is the rarest and the most delicate gift of all. The powers of observation must be exercised on the facts, and the various features of the problem must be noted; the process of reasoning must be employed on the observed relations and facts. But these processes are of an order far inferior to the keen intuition which can scent the direction in which a truth lies before it is fairly in sight. We must, however, continually emphasize the fact that an accurate intuition can only exist in a mind which is filled with facts which have been mastered, and relations which have been followed out, a mind in which past accomplishment has formed a medium in which the intuition may exist. We observe and reason only because we are finite. Only by this means can we present our results in form which is intelligible to others. A Supreme Intelligence would not need to observe nor to reason; It would know. The feeble approximation to this immediate knowledge which is possible to us is what I mean by the intuition. The great masters of science and mathematics have possessed this gift in large measure. We can all possess it in some measure. And it is our privilege and our duty to stimulate it in our students.

The power to appreciate the unity and harmony of mathematics is closely related to what I have been saying. Mathematics is full of relations which to the mind that is open to them are most suggestive of a unity which extends throughout the whole domain of science. In algebra, the fact that the solution of the quadratic equation affords all the types of number which are needed for all algebraic purposes is not only surprising, but indicates an economy in notation for our subject which is truly remarkable. In geometry the properties of the regular bodies, the sphere, the relations between the sphere and its circumscribing cylinders, and a host of other relations are most suggestive. Why should the square of one side of a *right* triangle exactly equal the sum of the squares of the other sides, that is, the simplest relation numerically be true, for the triangle which is simplest numerical relation be true, for the triangle which is simcoördination between the realms of number and of form, which

ought to be pointed out at every hand. Furthermore, when we pass to the physical world, why is the law of gravitation expressed in the very simple form of $F \backsim m/d^2$? Why should so many of the laws of nature seem to be expressed simply by our purely abstract numerical formulas? One cannot dismiss the question with the statement that we made our numbers to fit the laws, for the formulation of the laws followed centuries later than the adoption of the numerical notation. The numerical laws follow from the natural process of counting, a distinctly human device. The law of gravitation is not determined by our thinking about it, but is independent of our thought. This illustrates what I mean by the unity of science. What is simplest and most beautiful in the domain of pure mathematics, too often corresponds to the facts of nature to be accidental. I contend that it is our privilege to point out at every possible turn this coördination of number and form, of formula and physical law, of unity between the mind and nature. This is an experience of no mean moral value—to realize that our mathematical procedure is attuned to the harmony of the universe.

Columbia University.
New York City.

ALLIGATION.

By Willard A. Ballou.

In consideration of the almost absolute assurance with which the authors of arithmetics have omitted the topic of alligation from their texts for the past generation or two, I will confess that it is with some timidity that I am undertaking to show that this old-fashioned, illogical process greatly simplifies many of the calculations of a special but a rapidly increasing class of men, namely the industrial chemists.

You will better appreciate my timidity when I quote some of the not altogether enthusiastic references to this process by eminent mathematicians. Dr. Fench back in the eighties, after a long lifetime of work as a mathematician and educator made this apologetic remark concerning alligation:

"Alligation medial is largely applied to business affairs. Alligation alternate is more a mathematical curiosity; still there is a limited class of problems that have no other arithmetical solution."

Personally I am inclined to think that there are some problems that may be solved by alligation that have no other practical solution. Dr. D. E. Smith's appreciation may be correctly judged by the following quotation:

"A person *may* have an exercise in logic by studying alligation—merely indeterminate equations in an awkward, mediæval form."

Alligation, although it is held in favor in Germany, is rapidly and happily (as one author states) disappearing from the American and English text books.

I am willing to venture that one of the reasons why it remains a favorite topic in the German schools is because of the use that the chemists of that country find for this process, although I have very little direct evidence on this point.

Since many of the younger generation are not familiar with the processes of alligation I may be tolerated to explain and illustrate the more important principles.

To begin with let me take the following simple problem:

A cubic inch of Cooper's gold weighs 0.405 lb. What is its composition?

Cooper's gold is an alloy of copper and platinum. If we take the specific gravity of platinum as 21.5 and of copper as 8.83, we shall find that one cubic inch of platinum weighs approximately 0.775 lb. and one cubic inch of copper 0.318 lb.

Algebraic Solution.

$x =$ the volume of the copper in cubic inches present.

$y =$ the volume of the platinum in cubic inches present.

$$\begin{aligned} .318x + .775y &= .405, \\ x + y &= 1.000, \\ .318(1 - y) + .775y &= .405, \\ .318 - .318y + .775y &= .405, \\ .457y &= .087, \\ y &= .1904. \end{aligned}$$

That is there is 19.04 per cent. of platinum in the alloy.

x will then equal .8096 or 80.96 per cent. of copper in the alloy.

By Alligation.

	1.	2.	3.	4.	5.
Pt		0.775	0.370	1/370	87
	0.405				
Cu		0.318	0.087	1/87	370
					457

$$\frac{457}{87} \times 100 = 19.04 \text{ per cent. Pt, } \frac{370}{457} \times 100 = 80.96 \text{ Cu.}$$

In column 1 is the weight of the mixture.

In column 2 are the components or the simples.

In column 3 are the excess or lack in weight.

In column 4 are the medial ratios or the respective gains or losses referred to unity.

In column 5 are the quantities that should be mixed to give the required mixture, *i. e.*, 87 lbs. of a substance weighing 0.775 per unit volume when mixed with 370 lbs. of a substance weighing 0.318 lbs. per unit volume will give 457 lbs. of a mixture that will weigh 0.405 lb. per unit volume.

This of course readily checks, for:

$$87 \times 0.775 = 67.425$$
$$370 \times 0.318 = 117.66$$
$$457 \times 0.405 = 185.085$$

In this example one cubic inch was taken to facilitate the computation although naturally in practice the volume of the sample would not be any integral quantity, however its volume is determined with sufficient accuracy by measuring its displacement.

This example illustrates for us the first three important principles of alligation and although self-evident they are worth while stating.

Principal One.—The required average must be greater than some of the components and less than others.

Principle Two.—To produce a compound of a given average the losses caused by those components below the average must be balanced by the gain on those components above the average.

Principle Three. Any column of medial ratios may be divided or multiplied by any number without affecting the required average.

Even in this simple form involving two variables it saves the chemist much labor in certain common calculations of frequent occurrence. Following is given the two ordinary solutions of a simple problem of "a mixture with a common constituent" and after that the same problem solved to the same degree of accuracy by the method of alligation. I do this in detail that you may compare the amount of work involved in each method of solution. As the problems grow in complexity the greater becomes the power of alligation over the other methods until we arrive at a type of problem that has no other practical solution of which I am aware.

The silver from 4.22 grams of a mixture of AgI and AgBr weighed 2.11 grams. What is the weight of the iodine and of the bromine present in the mixture?

$X =$ weight of AgI present. $Y =$ weight of AgBr present.

$$X + Y = 4.22,$$

$$\frac{107.88}{234.8} X + \frac{107.88}{187.8} Y = 2.11,$$

$$
\begin{aligned}
Y &= 4.22 - X, \\
0.45945X + 0.57444Y &= 2.11, \\
0.45945X + 0.57444\,(4.22 - X) &= 2.11, \\
0.45945X + 2.42414 - 0.57444 &= 2.11, \\
0.11499X &= 0.31414, \\
X &= 2.7319 \text{ grams of AgI,}
\end{aligned}
$$

$$
\begin{aligned}
&2.7319 \times 0.540545 = 1.4767 \text{ grams of I,} \\
&4.22 - (2.11 + 1.4767) = 0.6333 \text{ grams of bromine.}
\end{aligned}
$$

This is a perfectly clear, logical solution, well understood by the student but laborious in execution, especially when the problem involves the solving of a system of four or five equations.

Solution by Proportion.

$2.11 \times 2.176493 = 4.5924$ grams of AgI $\equiv 2.11$ grams of Ag. $4.5924 - 4.22 = 0.3724$ grams, deficiency of the mixture due to the lighter atomic weight of bromin and proportional to the amount of bromine present.

$$\frac{126.92 - 79.92}{79.92} = \frac{0.3724}{X},$$

$$X = \frac{79.92 \times 0.3724}{47} = 0.633324 \text{ grams of Br.}$$

$$4.22 - (2.11 + 0.63324) = 1.47676 \text{ grams of I.}$$

This is a very good, brief method of solution for this type of problem but one not readily comprehended by students and in cases not so simple as this illustration liable to prove confusing.

Solution by Alligation.

Ag in AgBr from table 57.444 per cent.
Ag in AgI from table 45.945 per cent.
Ag in mixture (problem) 50.000 per cent.

AgBr	50.000	57.444	7.444	$\frac{1}{7444}$	4055	$\frac{4055}{114.99} = 35.264$ per cent. AgBr
AgI		45.945	4.055	$\frac{1}{4055}$	$\frac{7444}{11499}$	$\frac{7444}{114.99} = 64.736$ per cent. AgI

$$64.736 \times 4.22 = 2.73185 \text{ grams of AgI.}$$
$$2.73185 \times 0.54054 = 1.4767 \text{ grams of I.}$$
$$4.22 - (2.11 + 1.4767) = 0.633 \text{ grams of Br.}$$

It should not be passed over without notice that this method leaves the 35.264 per cent. of AgBr free to be used as a check, for the checking of this class of computation is of the utmost importance. By the other methods the check is as laborious, if not more so, than the solution itself.

It is desired to mix manganese ores containing 23 per cent., 41 per cent., and 47 per cent. of manganese respectively, so as to obtain an ore containing 39 per cent. of manganese.

1	2	3	4	5	6	7	8	9	10	11	12
39	23	16	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$	8	2	10	1	2	3
	41	2	$\frac{1}{2}$		$\frac{1}{2}$		16	16		16	16
	47	8	$\frac{1}{8}$	$\frac{1}{8}$		16		16	2		2
								42			21

Column 9 gives us the required mixture namely 23.8 per cent. of 23 per cent. ore, 38.1 per cent. of 41 per cent. ore, and 38.1 per cent. of 47 per cent. ore; but suppose the supply of 47 per cent. ore is limited so that it is not desirable to use so high a percentage of that ore. Divide column 7, the column containing the medial ratio in integral form which involves the 47 per cent. ore, by 8, writing the result as column 10; reproduce column 8 as column 11; and then column 12 will give another mixture which will produce a 39 per cent. ore, but one containing only 9.52 per cent. of the 47 per cent. ore. Both of these mixtures check, as well as an infinite number of other mixtures that might be found in the same way to conform to almost any limitations likely to be imposed by the commercial world. It is in this property that the power of alligation lies when it is applied to commercial problems.

From the above class of examples three other principles of alligation may be deduced:

Principle Four.—An original medial ratio in alligation alternate consists of two terms only.

Principle Five.—The sum of two or more medial ratios is itself a medial ratio.

Principle Six.—A medial ratio derived from other medial ratios by addition consists of a number of terms not less than two nor greater than the number of simples or components.

Samples of different lots of coal showed on analysis sulphur 2.4 per cent., 1.2 per cent., 1.7 per cent., 2.4 per cent., and 1.1 per cent.

Since coal containing more than 1.5 per cent. of sulphur forms excessive slag and burns out grate bars, it is desired to know in what percentage these different grades of coal shall be mixed so that they shall not average more than 1.5 per cent. of sulphur.

	1.1	4	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$				2	9	12				23	34.33 per cent.
	1.2	3	$\frac{1}{3}$				$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$				2	9	12	23	34.33 per cent.
1.5																	
	1.7	2	$\frac{1}{2}$	$\frac{1}{2}$			$\frac{1}{2}$			4			3			7	10.45 per cent.
	2.4	9	$\frac{1}{9}$		$\frac{1}{9}$			$\frac{1}{9}$			4			3		7	10.45 per cent.
	2.7	12	$\frac{1}{12}$			$\frac{1}{12}$			$\frac{1}{12}$			4			3	7	10.45 per cent.
																67	

This problem is very practical and shows how the medial ratios of each gain must be compared with the medial ratios of each loss when there are several of each, and how the derived medial ratios in integral form are combined by addition to give the proper quantities to be mixed.

As to the extent that the chemists may make use of the process I would mention the mixing the different normals to obtain the one desired, also in the dilution of chemicals in solution to any desired strength. It is very useful in mixing compound alloys, *i. e.*, in obtaining an alloy of desired composition from other alloys whose composition is known. In mixing alcohols or other liquids that show a contraction you cannot make use of the process without correcting for this contraction by the use of tables giving this contraction. However, you can easily change the specific gravity to the corresponding percentage by weight and then proceed without regard to the volume occupied by the mixture. I have often used it to advantage in the dilution to the desired strength of chemicals in solution. This is analogous to the only practical use I have found alligation put to in this country and that by the wholesale drug houses. They tell me that they could scarcely do business without it.

An order comes in for .78 morphine and they have in stock only .725 and .803 morphine. They work out the quantities by simple alligation and mix to the desired strength.

I do not know that I would advise the teaching of alligation to all classes of students, but I would strongly advise it for all people going into certain lines of practical work and especially for chemists. It would help them out of many of their difficulties and would facilitate certain operations that they now perform by tedious methods.

PRATT INSTITUTE,
BROOKLYN, N. Y.

THE PROVISIONAL REPORT OF THE NATIONAL COMMITTEE ON A GEOMETRY SYLLABUS.

By Howard F. Hart.

We have before us for discussion the Provisional Report of the National Committee of Fifteen on a Geometry Syllabus. The history of the formation of this committee and the results of its subcommittee on Logical Considerations have already been presented to this Association by Mr. E. R. Smith and printed in the Mathematics Teacher for June, 1910. When it was suggested to me that I discuss this report before you I hesitated because the form of this report, in which at times it appears to be the habit of the committee to propose a change, then debate the question and finally straddle the fence, does not lend itself to criticism, and also because in general this is a good report and the result of much work. I accepted in the hope that certain features of this report might be discussed and the Association take definite action with respect to them.

The report is divided into five sections as follows: (1) An historical summary of attempts to change the geometry course in France, Germany, Italy, England, and the United States, (2) Logical considerations, (3) Special courses, (4) Exercises, (5) Syllabus of propositions. I propose to omit any consideration of the first and third sections and only to make such references to the second section as are necessary. This should give us time for consideration of the larger things advocated by the committee. These I take to be:

I. The claims for geometry.

II. The attitude toward definitions.

III. The recommendation of informal proofs for some propositions now formally proved.

IV. The position with respect to the incommensurable case.

V. The recommendation on the distribution of exercises.

VI. The recommendation on the nature of exercises.

VII. The use of the concept of motion.

VIII. The use of algebraic notation in proofs.

IX. The geometrical construction of formulas.

X. The recommendation of the introduction of the trigonometric ratios.

XI. The syllabus proper.

After a warning against the dangers of dull formalism on the one hand and extreme radicalism in applications on the other the committee contends for geometry that it is primarily an exercise in logic and in such unusually simple setting that its results can easily be carried over into the lives of the students. In addition to the logical part is the training gained from accurate and precise thinking and the expression of such thinking together with the experience gained by contact with exact truth. Geometry also develops the idea of functionality, the power of space intuition and visualization, and is of course the basis of mensuration. In all these things geometry is an unique study in the secondary curriculum. We should all be glad that the committee obviously felt that there was no occasion or need for any excuses of any sort. To quote directly, "If we had to justify the position of any other subject in the curriculum history, rhetoric, geography, biology, etc., it is doubtful whether equally specific and cogent reasons could be found."

In the matter of definitions the recommendations of the committee can only lead to much good. In the first place it is recognized that certain terms cannot be defined in the strictly logical sense. In such cases the committee recommends that no so-called definition be attempted but that the concept be most carefully built up; but where definitions are possible and proofs depend upon them the definitions should be given. Also the committee is not in favor of definitions being taught other than at the point where they are needed. It regards the form of a definition as immaterial so long as it is as scientific as it should be for a secondary school.

The committee feels that it is not worth while to prove formally certain types of propositions. It gives among other examples, (1) All straight angles as equal, (2) Two straight lines can meet but once, (3) Polygons similar to the same polygon are similar to each other, (4) A straight line can cut a circle at most in two points. The committee admit the dangers involved in the carrying of such a thing beyond reasonable limits

and close their debate on the point as follows: "With all the experiments at improving Euclid the world has really accomplished very little except as to the phraseology of propositions and proofs; the standard propositions remain, and if geometry has any justification, apart from its kindergarten aspect (which requires but a short time), most of these propositions will continue to be proved, and should continue to be proved." It is obvious that a proposition which is an immediate influence from a definition can be best so derived on the spot, *e. g.*, in the second illustration it would probably be done somewhat like this: Two straight lines can meet but once, for if they were to meet twice they would coincide (straight line axiom) but since they are not coincident (given) they meet but once (negative-converse law). I assume that this is the kind of thing recommended and on that basis commend it.

Limits and incommensurable cases have long been a source of trouble. Constant, variable, limit, can be easily enough developed as concepts but the use of the Theory of Limits to prove incommensurable cases does not seem to be equally developable. So the committee recommended that commensurable cases only be required by any examining body. The present tacit understanding is unfortunate in that examiners cannot set a proposition which has the two parts. If this recommendation is definitely adopted it will end an uncertainty and be a relief to everybody.

In writing proofs the use of the symbolic or algebraic notation is advocated. It is claimed that the dangers will be obviated if the students be required to give the full equivalent in words in all oral work. My own experience, and I have been using an operative system of symbols in solid geometry for some years now (this experience was kindly printed by the MATHEMATICS TEACHER for September, 1910) makes me feel that the recommendation is very valuable and that the dangers can be obviated and no bad effects need result. The dangers involved are, however, very real ones. For an illustration let us consider a right-angled triangle. There are two propositions based on this figure which have to be carefully differentiated. They are (1) "The square on the hypothenuse equals the sum of the squares on the legs," (2) "The square of (the length of) the hypothe-

nuse equals the sum of the squares of (the lengths of) the legs." The tendency would be to make one symbol a^2 represent two such different things as a limited portion of a plane and a number; and, unless the teacher is everlastingly vigilant, that is just what will happen. In fact it happens in this very syllabus.

The committee recommended that in developing loci theorems the phraseology used be descriptive of a point moving under limiting conditions rather than of an assemblage of fixed points each satisfying the conditions. In my opinion there is no demand for this and I see no great advantage that would result from the change. For we all know that the point, which in a loci theorem we are either proving to satisfy the condition because it is on the line or conversely, is a fixed point. That it was moving before or will move later is not essential to the proof The committee base their recommendation on the fact that the student is already pretty familiar with motion as a concrete experience and that we should therefore use this experience. I doubt if it would prove very helpful. As in the well-known case of measurement, with which also the student is concretely familiar but thinkingly almost an entire stranger, so it will be here. The common experience of beginners in analytical geometry is further testimony in the same direction.

On the other hand the recommendation that the natural trigonometric functions of acute angles be introduced is at least feasible. That the students should, however, compute, or determine by approximate measurement, a two-place table of functions is not equally worth while in my opinion. On its pure mathematics side there is little (if anything) to be said for it. As an application it would be a good laboratory exercise in natural science. We must never forget that geometry is about the only course in the secondary curriculum which is ideal in the philosophic sense, nor must we ever adopt any procedure which will give it the air of a laboratory course in physical quantities. Personally I am opposed to having my students make their construction work an approximation process. As an illustration: "Given the line AB and an external point C to draw a perpendicular through C to AB." In this construction I have uniformly rejected any solution given in which the radius of the circle to be drawn, with C as a center, is not properly de-

termined. Finally, in very few schools indeed will there even be time for the introduction and proper use of the ratios only.

The recommendation that part of the exercises be placed under the theorems to which they are corollaries instead of all the exercises being grouped at the end of a section, must commend itself to all. This is current practice in the best books.

The recommendations on the nature of the exercises are to quote directly, (1) "There are several types of genuine problems, but many of the so-called real applications either are too technical to be within the grasp of the young beginner or represent methods of procedure that would not be followed in real life. Moreover, it should be remembered that the very limited time devoted to plane geometry renders it impracticable to introduce many of the applications that might be desirable if the time were not so restricted." Later it is recommended, "that a judicious selection of a reasonable number of abstract originals be made in order to leave time for an equally reasonable number of problems, particularly those with local coloring, stated in concrete setting." If the committee means what it says in both the quotations then it may be a consistent recommendation and descriptive of an ideal. But I think it only fair to say that pages forty-seven to fifty-four immediately following the second quotation, in which are set forth in detail sources of problems from architecture, decoration, design, together with a bibliography of the "real problem" propaganda would make one think that the committee actually recommends a maximum of "problems in concrete setting." I hope that on this one point at least the Association will take definite action.

I am in hearty accord with the suggestion that the geometric construction of equivalence formulæ be a part of a standard course.

We come now to the syllabus proper. Probably the first thing that will be noted is the way the theorems are printed. This was the means adopted to indicate the relative importance of the theorems. Those in heavy type are basal theorems of their group; those in italics are of considerable importance, but secondary to the first; those in roman type are still less important; while those in small type could be "omitted without serious danger." This is in my opinion a fundamentally helpful

and necessary improvement. An attempt to realize this is made by all good teachers. For another means to get this result compare Professor Peck's article (p. 54) in the MATHEMATICS TEACHER for December, 1910.

When we come to study the working out of this idea in this syllabus we meet some radically new points of view on certain theorems. Perhaps the most radical one is the reduction of inequality propositions to that class which can be "omitted without serious danger." Thus (1) If two sides of a triangle are unequal the opposite angles are unequal and the greatest angle is opposite the greatest side and conversely; (2) The exterior angle is greater than either interior angle not adjacent; (3) In two triangles having two sides respectively equal if the included angles are unequal the third sides are respectively unequal in the same order and conversely, (4) Of all the lines drawn from a point to a line, etc.; (5) In the same or equal circles the greater of two chords is nearer the center and conversely, etc., can be "omitted without serious danger." I am wondering why the committee adopted this. Was it to shorten the list some way, some how, so as to give time for "problems in concrete setting"? Or was it because the proof of the inequality propositions generally involve the proof of a location? If the latter case, are we to understand that the committee would only have us study such configurations as present no serious difficulties? A further study of this question, particularly in view of the theorems for solid geometry, makes us wonder still more, for there the inequality propositions are not printed in small type and cannot, as we know well, be omitted. Yet their proof requires the inequality theorems of plane geometry.

Continuing our study of small type theorems, we observe that the committee is willing to have omitted (1) the theorem which tells us when two circles intersect and is therefore the basis of constructions, (2) the construction of a tangent, (3) all propositions on the concurrent lines of a triangle, (4) the ways of proving lines not parallel by means of angles, (5) all propositions on the locus of the vertex of a fixed angle opposite a fixed line segment though this is one of the two great ways of realizing circle loci, etc., etc. The following are absolutely omitted, no reference being made to them in any way: (1) The theorems

of proportion as such; (2) lines perpendicular to intersecting lines; (3) the sum of the exterior angles of any convex polygon; (4) the line joining the midpoints of the sides of a triangle (though the general proposition is given, X. 3); (5) the midpoint of the hypothenuse of a straight triangle; (6) the inscriptible quadilateral; (7) the development of arcs from proportion (to be assumed as definition); (8) construction of a triangle equivalent to a given polygon, etc.; (9) harmonic division of a line segment, etc., etc.

In the syllabus of propositions for solid geometry we find among the theorems for informal proof the one on the intersection of two planes though this is an important locus theorem and should be proved as such. I assumed that the new axiom, "Two intersecting planes have at least two points in common," was given for that very purpose. So also we find here the proposition on the measure of the dihedral angle. From the point of view of its importance it should be in as heavy type as any proposition in the syllabus. And furthermore, it should be divided into the three customary propositions and proved in that way.

In the next section the committee takes the position that "Two straight lines are parallel to each other *if and only if* they are both perpendicular to some one plane." I might say that three planes in space can intersect in three lines and that, if they do, these three lines must either be concurrent or parallel. This can be demonstrated without any reference to being perpendicular to any plane whatsoever. And it is a sufficient basis for showing that lines parallel to the same line are parallel to each other.

I might continue this line of criticism to much greater length but will stop with the observation that this syllabus will be famous, or otherwise, for what it omits.

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STANDARDIZATION OF ARITHMETICAL WORK.

By W. L. Vosburgh.

For an understanding of the scope of this discussion, it seems necessary to define at the outset the term "standardization" as herein used. "Standardization of arithmetical work" is to be considered from the point of view of modern educational psychology, viz., that its basis be the facts of arithmetical experience rather than mere opinions regarding the same. This view implies the possibility of establishing objective scales for measuring arithmetical abilities.

A word as to the distinction between *subjective* and *objective* scales of measurements for mental abilities seems necessary and essential at this point. The explanation of this distinction is taken from Thorndike's "Educational Psychology," page 10: "A measurement of human nature to be useful for our purpose must identify the *amount* in question for any competent thinker, just as a useful description must *identify* the *object* in question To do this it must be objective, that is, free from individual caprice, so that any competent person making the same measurement would get the same result. (Not of course exactly the same. There is a personal equation in even the most objective measures, such as the length of this line ————. If they measured it to thousandths of a millimeter, competent observers would not get the same result, except by chance. Nor would the same observer in several independent measurements. The ultimate distinction between objective and subjective is simply that in the former sort of measurements competent observers use very nearly the same criteria and, though independent, agree very closely, whereas in the latter they use very different criteria and, if independent, agree only roughly. Objective measures may be defined as measures which competent observers could repeat and verify or reject, and subjective measures as measures which they could not so repeat and verify.)"

The purpose of this paper is to arouse an interest in a scientific study of the teaching of arithmetic, hoping thereby to encourage the teachers of arithmetic to measure the results of their

teaching objectively—a more definite and scientific way than is possible by means of the ordinary form of test or examination, and to secure their co-operation in the work of establishing objective scales of measurement for all arithmetical abilities.

Desirability of Objective Standards.

The desirability of objective standards in all school work, and particularly in arithmetical work, is I think obvious to all of us; the problem with which we are concerned is the possibility of establishing such standards in arithmetical work. Before considering the latter however I will quote from two authorities as to the former.

Dr. Hanus in his paper on "Our Chaotic Education" which he read before the Department of Superintendence of the N. E. A. at its meeting in February, 1902, said as follows: "Now, just as we have not organized and adequately assimilated a generally accepted educational doctrine, so we are without a body of recorded educational experience. Results actually achieved and collectively presented constitute a force that is capable of sweeping away superficial criticism or paralyzing scepticism on the one hand, and meddlesome interference and impatient clamoring for premature results on the other. Isolated successes have been advertised, to be sure, and failures, more or less obvious, have sometimes been frankly confessed, and sometimes unwisely suppressed. But in neither case have we had an orderly presentation of both successes and failures over a wide area. We have had plenty of experiments; indeed, as I have intimated, our whole educational activity for nearly a generation has consisted of experiments. Just as every educational theorizer has worked by himself without taking due account of the labors of his fellow-workers in the same field, so every superintendent has pursued his way, apparently in blissful indifference to what his fellow-superintendents were doing, multiplying instances and varying conditions *ad libitum*. How is it possible to extract any confirmation of alleged results from such a heterogeneous procedure? And we never can get such confirmation until we abandon our absurd extreme of individualism in these experiments and work together for the attainment of the same ends. . . .

"Briefly, then, we must organize our educational experience just as we must organize our educational doctrine, if we are to make real progress."

He then gives two or three illustrations of what he means; the one of interest to us is as follows: "Suppose the conditions to be something like this: Five hundred or a thousand pupils in each city to begin the study of arithmetic in the first year, and a similar number to begin it in the second year, and a third similar group to begin it in the third year of school. At the end of the sixth year of school compare the attainments of the three groups of pupils. Would not the conclusions reached by such an experiment have a convincing value which no amount of assertion beginning 'in my schools,' or 'so far as my experience goes,' or 'I believe,' or 'in my opinion' could possibly have? . . . I do not believe that such co-operation is impossible. Why should it be? Experiments similar to those suggested are everywhere in progress; co-operation in large enterprises of all kinds is possible. Why should it be impossible in education only?

"Under such circumstances we could face the teaching profession and the general public with facts, instead of opinions. The enormous difference between the weight of these two very different things in educational affairs still remains to be experienced. . . ."

The second authority on the desirability of objective standards in school work, Dr. E. L. Thorndike, the inspiring genius of the quantitative study of education in American today, said in 1904: "The study of education is beginning to be quantitative. We are no longer satisfied with vague arguments about what this or that system of administration or method of teaching does, but demand exact measures of the achievement of any system or method or person. We are becoming properly disgusted with the one-sided book-keeping which only takes account of the dollars spent, and neglects the debit side, the income in knowledge, habits, power, zeal, and ideals.

"This ambition toward an exact objective measurement of the results of educational endeavor is a symptom of healthy scientific fervor and also of common-sense wisdom. It is in every respect and for every reason right that the same painstaking

precision should be applied to the study of human welfare that is now given to the study of Jupiter's satellites or to the petals of the daisy. It is right, too, that to a general faith in the efficacy of systematic education there should be added a cold-blooded inquiry into just what comes of it all. No one possessed of either science or sense will deny the value of successful quantitative study of school work.

"The complexity of the phenomena of education is obvious and need not be discussed here. The lack of commensurate units in which to measure intellectual and moral qualities has been by far the greatest barrier to quantitative science, all the more so in education because we have been unconscious of the lack. The units at hand are defective in that they are subjective, depend on individual caprice instead of universal agreement, are not commensurate, and are reckoned from unknown zero-points. In even so simple a matter as ability in arithmetic, the measure to be given to any individual is not an easy affair. In the first place, the marks will vary somewhat with the individual who examines the examples done; secondly, the credit to be given for any example as compared with any other is unknown; and, next, even if the test comprised twenty examples known to be of equal difficulty and to have been marked impartially, the fact that one boy scored 14 and another 7 need not mean that the former is twice as far above zero in arithmetical ability as the latter."

Possibilities of Objective Standards.

Our problem, the possibility of standardizing arithmetical work, is summed up by Dr. Thorndike in the following: "To have in education the real benefits of quantitative science, we must spend arduous years in devising, testing, and standardizing units of measurement, in searching for convenient arbitrary zero-points, and eventually for real zero-points, and in determining the sources and amounts of the errors of measurement. It has taken many centuries and the labors of many gifted men to develop the present system of physical measurements, and the task is far harder in the case of intellectual and moral facts.

That the adequate measurement of intellectual and moral facts is difficult does not, however, at all imply that it is im-

possible or that inadequate and erroneous measurements are not useful. . . . Inadequacy and error are relative terms. It is a great advance to regard the sun as ten times farther away from the earth than the moon is, if the previous belief was that it was only half as far. . . . Only the ignoramus regards any measurement as perfect. All are approximations, and a very rough approximation may be far more accurate than previous measurements."

We are fortunate in having, at the present time, three very definite and specific contributions to the scientific study of arithmetical abilities. These are the studies by Rice ('02), Stone ('07), and, Courtis ('10 and '11), respectively.

Dr. J. M. Rice, formerly editor of *The Forum,* may be regarded as the man in this country to take the initiative in an attempt to study scientifically the actual achievements of the schools in the school arts (his studies investigating the teaching of spelling, arithmetic, and language), with a view to establishing just how much time can be saved by a suitable restriction and selection of subject matter.

Dr. Rice in 1902 measured the arithmetical abilities of some 6,000 children of grades 4, 5, 6, 7, and 8, in eighteen different schools, in seven different cities. His tests for each of these grades and a discussion of the results attained by the various systems may be found in the *Forum,* Vol. 34.

We are concerned here primarily with: (1) the abilities tested, (2) the variation in the results attained within the systems and between the systems, and (3) the actual contribution of his study to the establishment of objective standards in arithmetical work.

The test itself consisted of eight examples for each grade. The examinations were given to the pupils of the fourth, fifth, sixth, seventh, and eighth grades. They were not given below the fourth grade because he aimed to discover what children are able to do on leaving school. For the purpose of studying the growth of mental power from year to year, some of the problems were carried through several grades. Thus, of the eight questions for the fourth grade, five were repeated in the fifth, and three in the sixth, etc. Moreover, this repetition enabled him to see not only how the results in the fifth and sixth grades, in

regard to certain problems, compared with those of the fourth in the same school, but also how the results in the fourth grade of some schools compared in these examples with those of the fifth and sixth grades of others, etc. The subject matter of each test was such as to attempt measure, at the same time, ability in both reasoning and fundamentals.

The range of variations in the results obtained on these tests was very wide, viz., in the seventh year the class averages range from 8.9 per cent. to 81.1, and in the eighth year from 11.3 to 91.7. The averages for schools taken as a whole varied between 25 and 80 per cent.

The general average for all schools examined was, in round numbers, 55 per cent., made up as follows: 60 per cent. for the fourth year, 70 per cent. for the fifth, 60 per cent. for the sixth, 40 per cent. for the seventh, and 50 per cent. for the eighth. In view of what the satisfactory schools have shown, Dr. Rice thinks that 60 per cent. for the fourth grade, 70 per cent. for the fifth, and 60 per cent. for the sixth are reasonable expectations. However, 40 per cent. for the seventh grade and 50 per cent. for the eighth are too low, as these figures are not at all representative of what the successful schools have been able to accomplish, but result from the fact that, in the majority of instances, the seventh and eighth grades were lamentably weak. As the average for the seventh grade of the five successful schools was 61.4, and that of the eighth grade 77.2, he thinks that less than 50 per cent. for the seventh year and 60 per cent. for the eighth should not be regarded as satisfactory.

Dr. Rice says: "In suggesting a standard, it is, of course, understood that the figures mentioned in the last paragraph would only be applicable to an examination whose degree of difficulty were the same as my own. Teachers desirous of knowing how their pupils compare with those of other schools could try the questions as I have given them, or, if they feared that the publication of the problems had diminished the value of the test, they might change the figures without altering the degree of difficulty. However, in due course of time there ought to be no difficulty in establishing standards in arithmetic with mathematical precision. This may be quite readily done by selecting types of examples and determining by research

what percentage ought to be obtained on each of them by the class for which they are intended. When this point has been reached, a standard will also have been fixed for a combination of examples of various degrees of difficulty."

In addition he gives us the following conclusions in regard to twelve possible factors of school work. These are based upon the facts revealed in the results attained by the different school systems. His conclusions are briefly summarized, by Stone, as follows:

(1) *Home environment:* "As in spelling so in arithmetic, this mountain, on close inspection, dwindles down to the size of a molehill"; (2) *Size of classes:* "No allowance whatever is to be made for the size of the class in judging the results of my test"; (3) *Age of pupils:* "This factor can be held accountable for the difference shown to only a slight degree if at all"; (4) *Time of day:* "This likewise is not a determining factor"; (5) *The time devoted to arithmetic in the school:* "A glance at the figures will tell us at once that there is no direct relation between time and result"; (6) *The amount of home work required:* "This fact is also denied to be a controlling one, as the highest five schools had practically abandoned home work, while the lowest ranking city required most"; (7) *Method of teaching:* In the schools that passed the tests satisfactorily no special method had been in use; (8) *Teaching ability:* "Teachers of most successful schools had no better ability than those of unsuccessful schools"; (9) *Course of study:* By a curious line of argument Dr. Rice reaches the conclusion that the course of study is not a factor to be considered because the tests were fair to pupils who had been taught by all courses of study; (10) *Superintendent's training of teachers:* This was as much in vogue in localities that did poorly as in those that did well; (11) and (12) *Superintendents' establishing of standards and testing for results:* These Dr. Rice concludes to be the large and controlling factors.

In conclusion Dr. Rice says: "By reason of the high percentages obtained in certain schools, laboring under ordinary conditions, we must accept as a fact that nearly all children can be trained to solve any ordinary problem in arithmetic, based upon principles they have studied. Consequently, if the normal child is not reasonably proficient in that branch, as far as he has advanced in it, the fault is not his."

The second contribution to the quantitative study of arithmetical work is that of Dr. C. W. Stone made in 1907. The results of this study were published in 1908 as Vol. 19 of Columbia University Contributions to Education (Teachers College Series) entitled "Arithmetical Abilities and Some Factors Determining Them." This study took account of, and profited by, that made by Dr. Rice. Its improvements over the former are in the data used, and in more refined and scientific methods of gathering and handling the same.

"The central purpose of this study" (to quote the author), "broadly stated, is to make one more contribution to the exact knowledge of the relation between distinctive educational procedures and the resulting products."

The *method* of this inquiry may be briefly characterized as the application of the statistical method to mental measurements. The three principal measurements made are the arithmetical abilities of 6A pupils, the time expended, and the course of study materials used in securing these abilities.

The sources of data were 26 school systems in six different states from Indiana to Massachusetts. Dr. Stone visited and personally gave, under conditions as nearly identical as possible, two tests, one in fundamentals and one in reasoning, to 3,000 6A grade children. The time for the test in fundamentals was set at 12 minutes; for reasoning at 15 minutes, each test being purposely too long to be finished by any pupil within the time limit (14 examples in fundamentals, 12 in reasoning).

The main purpose of the test in fundamentals was to determine ability of 6A grade pupils in addition, subtraction, multiplication and division. The main purpose of the test in reasoning was to determine the ability of 6A grade children to reason in arithmetic. The test in fundamentals was so arranged as to enable the pupils to meet the main difficulties in the first six problems; in both tests the problems were arranged in order of difficulty with the less difficult first.

The variations in abilities between the systems in the grade tested (6A) were as follows: In reasoning the points scored varied from 356 to 914, with an average deviation of 112, the median being 551; in fundamentals from 1,841 to 4,099, with an average deviation of 421, the median being 3,111. Translated

this means that if we take the efficiency of the school making highest score in reasoning as 100 per cent. the median represents an efficiency of 60 per cent., the lowest of 39 per cent.; in fundamentals the median score represents an efficiency of 76 per cent., while the lowest 45 per cent. The variability among individuals within a system was even greater than that among systems. Among 500 pupils chosen at random from four representative public school systems: In fundamentals they varied from 0 to 15.2: 9 out of 500 failed to solve any example in the reasoning test: The median score was about 6: 33 made score of 2 or less. In one system 15 out of 100 pupils made no score in subtraction and 29 made one each. In general, pupils varied most in division, next in reasoning and about equally in addition and multiplication. In regard to the fundamentals, from their coefficients of correlation, he tentatively states the following: "that the possession of ability in addition is the least guarantee of possession of ability in others; that the possession of ability in multiplication is the best guarantee of the possession in others; and that this probably means that multiplication is like addition on its mechanical side and like division on its thinking side. Hence if one wished to measure abilities in fundamentals by a single test, one in multiplication would be best; and a test in division would probably be the best single measure of arithmetical abilities." As to the lack of correspondence between serial standings in time-cost and serial standing in abilities of systems we find the system devoting the smallest per cent. of time to arithemtic (7 per cent.) was third in average standing of comparative achievement; the system devoting largest per cent. of time to arithmetic (22 per cent.) was twelfth; the one devoting 14 per cent. was lowest of all.

Contributions.

His conclusions may be summarized as follows: (1) That the general method of this research is a means by which hypotheses are to be tested and opinions to become facts. (2) That *diversity* sums up the findings of this study, as shown by: (I.) A variability of scores among systems of 356 to 914 points, with an average deviation of 112 in reasoning; and 1,841 to 4,099, with an average deviation of 421 in fundamentals; (II.) A variability of mistakes among systems of 14.5 per cent.

to 4.7 per cent. of all the steps attempted in addition, and 45.1 per cent. to 14.4 per cent. of all the problems attempted in reasoning; (III.) A variability of 507 to 1,854 week-minutes with an average deviation of 222 week-minutes in time expenditure, without home study; and 507 to 2,179 week-minutes with an average deviation of 269 week-minutes, including home study; (IV.) A variability in average ratio of time to abilities of from 2.26 to .64; (V.) A difference in course of study excellence which can hardly be put in words. (3) The greatest need shown by the research is standards of achievement. That the great variability herein shown would exist if school authorities possessed adequate means of measuring products is inconceivable; and it is believed that the present study will help standardize the work in arithmetic for the first six grades. (4) Finally that there is no *one* factor that produces abilities, there is no single *summum bonum* in teaching arithmetic.

The third contribution to the quantitative study of arithmetical work is that of S. A. Courtis, an account of which may be found in Vols. 10 and 11 of *Elementary School Teacher;* the latest report of the progress of his study may be found in the *Elementary School Teacher* for November, 1911. In this latest study he has secured the co-operation of teachers in England and Germany as well as in the United States. Mr. Courtis began his study of the measurement of growth and efficiency in arithmetic, in 1908, in the school with which he is connected (The Liggett Home and Day School of Detroit, a private school for girls). His ambition was to find the place of his school among the 26 school systems reported by Dr. Stone. The results secured by his school on the Stone tests and an analysis of the same are given in the *Elementary School Teacher,* Vol. 10. In speaking of the results the test yielded he says: "It has shown the need for greater knowledge of what is actually taking place in the child's mind as he passes through grade after grade. It has made plain the uncertainty of the product of the present educational system as well as the value of a complete and many-sided training in the early grades if there is to be great ability in the later grades. More than all else it has proved, conclusively to the writer at least, that it is practicable to measure, not only the general condition of arithmetic teaching throughout

a school, the growth in ability and efficiency from grade to grade, the defects and needs of any one grade or individual, but the *effects of changes in method or procedure as well.* By a series of tests, through a number of years, it ought to be possible to build up a real science of teaching and to determine by strict experimental methods the truth or falsity of any educational hypothesis." With this idea in mind Mr. Courtis devised, the following year, 1909, eight tests designed to measure the change produced by a year's regular work under existing conditions in order to have some standard by which to measure the effect of future changes in method. Because of the complexity of the problem, the scope of the investigation was limited to the simplest work with whole numbers in the four fundamental operations. The specific abilities to be measured by the tests were: (1) ability to give a ready response in the case of any of the elementary combinations in the four operations, (2) ability to recognize a situation in a problem as calling for the use of a certain operation, (3) ability to borrow and carry, and (4) the ability to copy correctly the figures of a given example or problem. Accorcdingly provision was made for the testing of each of these as well as the general abilities involved in working abstract examples and concrete problems.

The subject matter of the tests, eight in number, was as follows:

Test No. 1, Addition,
Test No. 3, Multiplication,
Test No. 2, Subtraction,
Test No. 4, Division,
(Combinations, 0–9),

Test No. 5, Copying figures (rate of motor activity),

Test No. 6, Speed reasoning (simple one-step problems),

Test No. 7, Fundamentals (abstract examples in the four operations),

Test No. 8, Reasoning (two-step problems).

The nature of the investigation made it essential that the tests should be given to all grades from the lowest to the highest under identical conditions.

The quantitative studies of the teaching of arithmetic in his school during years 1908–09, 1909–10, showed Mr. Courtis the possibilities of measuring accurately the changes produced by

the teaching effort. Clearly defined standards alone were lacking. His latest study therefore deals with an attempt to determine "standard scores in arithmetic" and may be found in *Elementary School Teacher,* November, 1911.

In this study he has already secured the co-operation of some 60 or 70 different schools, in 10 different states, from Kansas to New Hampshire. More than 18,000 sets of his tests have been distributed by him and the co-operation of teachers in England and Germany in this study is already assured. Last June more than 9,000 individual scores, from 60 to 70 schools, were returned for tabulation.

The subject matter of these tests has been considered; as to the variations in abilities shown among the systems we have: Of 1,401 eighth-grade children measured as to their knowledge of the multiplication tables, 2 could write answers at a rate of but 5 answers a minute, 17 at the rate of 15 answers a minute, 180 at 25 per minute, and so on, the average for the entire group being 43.

It is particularly to be noted that the wide range of the distribution is caused not so much by differences between schools as by great variation in the abilities of individual children. The amount of variation in the grade averages of different schools does not equal the variation in the scores of the members of any one class.

The fact shown is a constant feature of the returns for all the tests and grades, and can only mean that the differences in the abilities of individual children are greater factors in determining relative rank in school work than all the differences in abilities of teachers, courses of study, or methods of work combined. The result is that the grades in all the schools overlap to an extent well nigh incredible.

Courtis's contribution to the standardizing of arithmetic is significant in that he suggests standard scores for the abilities tested, for each grade from III. to IX. inclusive. Such standard scores may be the average scores obtained on each test by all pupils tested in each grade. These are given in Table III. of the November article, p. 133; or a standard score may be derived as follows: "For each test an eighth-grade score was selected such that it was equalled or exceeded by 30 per cent. of the

eighth-grade children measured. This score was plotted and a smooth curve drawn, having the same general form as the average curve and coinciding with it in the lower grades. The scores for each of the other grades were then determined from the graph." Standard scores derived on this basis are given in Table IV., p. 135. Either of these standard scores may be used by teachers as definite objective standards toward which to work, *e. g.*, "at the end of a year's work, an eighth-grade child should be able to copy figures in pencil on paper at the rate of 117 per minute; to write answers to the addition combinations at rate of 63 per minute, subtraction, multiplication and division 49 per minute, etc., etc. At the present time 70 per cent. of the eighth grade cannot meet these standards but 3 per cent. of the fifth grade can.

A review of each of these three studies has been made with the hope of enlisting the sympathies of those assembled here in this movement tending toward the standardization of arithmetical work, and of making it apparent that as a foundation for such work we already have accumulated a well-defined and important body of facts of arithmetical experience. The value of these to all future quantitative studies of arithmetical abilities cannot be overestimated.

As to the part that each one of us may take in this movement, we may consider any one or more of the following suggestive lines of work:

1. That the tests and examinations now given be used in a scientific way and be made to be careful scientific measures of changes in our pupils. Dr. Thorndike says that "if an examination, instead of being a hasty, subjective selection of questions graded still more personally (and alas, how hastily), were made a serious educational measurement, the examination papers of a year would give us a large start toward knowledge of what (science) teaching actually does."

2. That we make use of the tests devised by Rice, Stone, or Courtis, as objective measures of achievement of our pupils and objective measures of the results of the teaching effort.

3. That there is an opportunity for co-operation with Mr. Courtis at the present time in his study, recently undertaken, to determine accurately the proper amount and form of drill re-

quired to produce the standard abilities suggested, and similar determinations of standard yearly growths.

4. That other and similar tests be devised for measuring objectively specific abilities in arithmetic and the results of our teaching.

In conclusion, a sample test, designed by myself to measure the ability of entering Normal students in addition will be submitted for your inspection.

BROCKPORT, N. Y.

NEW BOOKS.

Text-book on the Strength of Materials. By S. E. Slocum and E. L. Hancock. Revised Edition. Boston: Ginn and Company. Pp. 372. $3.00.

The second edition of Slocum and Hancock's "Text-book on the Strength of Materials" constitutes a thorough revision of the original text. In making this revision the aim of the authors has been twofold: *first,* to keep the text abreast of the most recent practical developments of the subject; and *second,* to simplify the method of presentation so as to make the subject easily intelligible to the average technical student of junior grade, as well as to lessen the work of instruction.

Besides correcting the errors inevitable to a first edition, special attention has been given to amplifying the explanation wherever experience in using the book as a text has indicated it to be desirable. This applies especially to the articles on Poisson's ratio the theorem of three moments, the calculation of the stress in curved members, the relation of Guest's and Rankine's formulas to the design of shafts subjected to combined stresses, etc.

Considerable new material has also been added. To facilitate numerical calculations a set of tables has been placed at the beginning of the volume. These include an extensive table of physical constants, a table of the properties of various geometrical sections, tables of the properties of various standard rolled sections, tables of moments of inertia and section moduli of rectangular and circular sections for a wide range of dimensions, tables of functions of angles, common logarithms of numbers and conversion from the common to the natural system, and vice versa, and a table of bending moment and shear diagrams for beams under various loadings and methods of support, giving the maximum deflections and other properties.

In Part I. the most important additions are articles on the design of re-enforced concrete beams, shrinkage and forced fits, the design of eccentrically loaded columns, the design and efficiency of riveted joints, the general theory of the torsion of springs, practical formulas for the collapse of tubes, and an extension of the method of least work to a wide variety of practical problems. This last includes the derivation and application of the Fraenkel formula for the bending deflection of beams, and also a simple general formula for the shearing deflection of beams, never before published.

Nearly one hundred and fifty original problems have also been added to Part I. These problems are designed not merely to provide numerical exercises on the text, but have been selected throughout with the specific purpose of emphasizing the practical importance of the subject and extending the range of its application as widely as possible. Many of them are practical shop problems, brought up by students in the Co-operative Engineering Course.

In Part II. the recent advances in the manufacture of steel have been given special attention, including the properties of vanadium steel, manganese steel, and high-speed steel. Re-enforced concrete has also received a more adequate treatment, and the chapter on this subject has been thoroughly revised and modernized. The chapter on timber has also received an equally thorough revision, and considerable material on preservative processes has been added.

Riverside Educational Monographs. Edited by HENRY SUZZALLO Boston: Houghton Mifflin Company. 35 cents each.

Individuality. By E. L. THORNDIKE.

Few teachers recognize sufficiently the importance of individuality in their work. The tendency is to bring all the members of a class to a common level, not only the same level of attainment, but to the same mental types and qualities. The author in this little volume points out the various individual differences and the causes which influence them.

Teachers who read it will find much of interest and profit and will have a truer conception of what they should accomplish.

Education for Efficiency and the New Definition of the Cultivated Man. By CHARLES W. ELIOT.

These two addresses treat the same subject from two different points of view—culture and efficiency—and should do much to give teachers a better conception of standards and ideals in education. Dr. Eliot's long experience in educational work and his standing among teachers will insure for this book a wide hearing. There is much sound judgment and wisdom condensed here in small space.

The Teacher's Philosophy in and out of School. By WILLIAM DEWITT HYDE.

Much has been said during the immediate past concerning the teacher's knowledge of subject matter and of his better understanding of the child and social life. While no one will deny the importance of these considerations it seems possible that in giving them emphasis the teacher has been almost forgotten. His individuality, his culture, and his efficiency have too often been neutralized by prescribed conditions and at the present time there is great need of a philosophy of teaching, a philosophy which takes into consideration "the teaching personality and the teaching life." Such a philosophy President Hyde gives in this volume—one which every teacher will do well to read and ponder.

How to Study and Teaching How to Study. By F. M. MCMURRY. Boston: Houghton Mifflin Company. Pp. 324. $1.25.

Professor McMurry has in this volume given a rather careful exposition of the nature of study and its principal factors, and their relation to children. In as many chapters he treats of the following eight factors in study: Provision for Specific Purposes, The Supplementing of Thought, The Organization of Ideas, Judging of the Soundness and

General Worth of Statements, Memorizing, The Using of Ideas, Provision for a Tentative rather than a Fixed Attitude towards Knowledge, Provision for Individuality.

There is perhaps no one point in school work so neglected as that of learning how to study. Every one is left to stumble into methods and habits of his own, which for the most part is conducive to anything but economy of time and efficiency of effort. The author has done a good service in giving this discussion.

Elementary Trigonometry. By F. T. SWANWICK. Cambridge: The University Press. Pp. 258. $1.25 net.

The first chapter of this book, which is on Approximate Arithmetic, is rather an innovation from the American standpoint. Such work if complete enough is of value and should be taken up somewhere, but an algebra would seem to be the more natural place to look for it.

The author defines: "The sine of an obtuse angle is equal to the sine of the supplementary acute angle." "The cosine of an obtuse angle is equal to the cosine of the supplementary acute angle." By means of these he has simple means of proving the formula for $(A \pm B)$ even when A and B are obtuse.

The book on the whole seems to have been carefully written and presents many excellent features.

Plane Geometry. By C. A. HART and D. D. FELDMAN. New York: American Book Company. Pp. 311. 80 cents.

This is a geometry of the usual type, as far as its general makeup is concerned, but it is well arranged typographically and contains some excellent features, for example, a very complete summary of the formulas of plane geometry.

It conforms to modern usage in its choice of propositions, and seems to have a very full set of well chosen exercises. There are also included some interesting references to important historical material.

One unusual feature is the definition of a plane figure as including both the boundary and the portion of the plane inclosed.

The Twenty-Seven Lines upon a Cubic Surface. By ARCHIBALD HENDERSON. Cambridge: The University Press. Pp. 100. $1.50 net.

The fact of a definite number of straight lines lying on the cubic surface seems to have been discovered by Cayley in 1849, while Salmon determined the number as twenty-seven. In 1869 Wiener constructed a model of the surface with the twenty-seven real lines lying on it. In this memoir the author gives "a general survey of the problem of the twenty-seven lines, from the geometric standpoint, with special attention to salient features: the concept of trihedral pairs, the configuration of the double six, the solution of the problem of constructing models of the double six configuration and of the configurations of the straight lines upon the twenty-one types of cubic surfaces, the derivation of the Pascalian configuration from that of the lines upon the cubical surface with one conical point, and certain allied problems."

NOTES AND NEWS.

On February 3 eleven teachers met in Pittsburgh and reorganized the Pittsburgh Section. There were in all twenty-one enrolled, of which eighteen were new members and with this enthusiastic start we predict a vigorous and prosperous section. Mr. W. F. Long, of the Central High School, was elected chairman, and Mr. Arthur C. Baird, of the Fifth Ave. High School, was elected secretary.

The Committee on Bibliography of Mathematics for Secondary School Teachers authorized at the Annual Meeting has been appointed as follows: Eugene R. Smith, Polytechnic Preparatory School, Brooklyn, N. Y., Chairman; William H. Jackson, Manchester, England; Daniel Pratt, Syracuse University, Syracuse, N. Y.; Clarence P. Scoboria, Polytechnic Preparatory School, Brooklyn, N. Y.; George Alvin Snook, Central High School, Philadelphia, Pa.; George F. Wilder, Erasmus Hall High School, Brooklyn, N. Y.

The plan of work has been made out, dividing the field as follows: (1) Publications of Associations and Colleges; (2) Publications of the State and National Educational Departments; (3) Articles in Educational Magazines; (4) Books and Pamphlets issued by Publishing Houses; (5) Foreign Publications most likely to be of use.

The Spring Meeting of the Association is to be held on April 6 at Syracuse University, Syracuse, N. Y. A good program is being provided and a large attendance expected.

The American Federation of Teachers of the Mathematical and the Natural Sciences held its annual meeting at the New Willard Hotel in Washington on December 27.

The association composing the federation reported concerning their activities during the past year, and the reports of committees were considered as follows:

Action on the recommendations in the report of the Committee

on College Entrance Requirements was postponed for one year, with the understanding that the various associations were to take action on it in the meantime, and were to report their decisions to the federation.

The National Geometry Committee report was approved as a report of progress. The chairman, Dr. Slaught, reported that a preliminary report would soon be distributed to all members of the federation who were engaged in mathematics teaching, as well as to such other teachers as were interested. An edition of 5,000 copies will be published, the expense being borne by the National Educational Association.

The amendments reorganizing the council by limiting the representation of each association to one member were adopted, as follows:

Section 5. Each association shall have one delegate on the federation council, this delegate to cast one vote for every fifty members of the association he represents, but to have at least one vote. The delegate may be chosen in any way decided upon by his association, shall hold office for three years, or until the appointment of his successor, and shall be eligible for re-election. In case of a vacancy by death or resignation, the association in question must at once appoint a successor.

Section 7. The duty of a delegate shall be to keep the secretary of the federation informed as to the activities of his association, and to represent the interests of his association at every meeting of the council. If for any reason he cannot attend a meeting, he shall be responsible for being represented by a properly accredited proxy.

The associations have been asked to appoint these representatives at once, and it is hoped that the new council will soon be in full working order.

The Treasurer reports as follows:

Receipts.

Balance from 1910	$ 85.60
Dues from 12 associations	179.20
	$264.80

Expenditures.

Stationery and stamps	$ 8.50
Printing and mailing	70.52
National Geometry Committee	100.00
	$179.02
Balance	85.78
	$264.80

One association, The Association of Biology Teachers of New York, has resigned from the federation, and two associations with an approximate membership of 150 have not yet paid their dues for the year.

A committee of teachers of physics, J. A. Randall, Pratt Institute, Brooklyn, Chairman; W. R. Pyle, Morris High School, New York City, W. A. Hedrick, McKinley Manual Training School, Washington, D. C., G. A. Works, Madison, Wisconsin, P. B. Woodworth, Lewis Institute, Chicago, has been appointed "to co-ordinate new apparatus and new teaching content with the present secondary school physics course."

Mr. Randall is chairman of a similar committee of the New York State Science Teachers' Association, and committees to co-operate in this work have already been appointed by the Physics Club of New York and the New Jersey State Science Teachers' Association. It is hoped that the National Educational Association will decide, at next summer's meeting, to be a partner in this undertaking, as it has been in the work of the National Geometry Committee.

The plan of work for the committee is to have each member act as chairman of a local committee, which shall investigate conditions in its territory, collecting data as to new apparatus and improvements in courses to be submitted to the general committee, and giving to the instrument makers plans for whatever apparatus seems worth while. The general committee will probably form a new definition of the "Physics Unit" to correspond with what they find to be the most improved usage in the subject, and will perfect machinery by which every physics teacher in the country can secure the most improved forms of equipment.

On Thursday morning, at a joint session with Section L of the A. A. A. S., the members of the Council listened to addresses

by Professor C. W. Moore, of Harvard, Professor A. L. Jones, of Columbia, and Dean J. R. Angell, of Chicago University, on the new systems of admission to these colleges.

EUGENE R. SMITH,
Secretary.

POLYTECHNIC PREPARATORY SCHOOL, BROOKLYN.

WORLD Peace Foundation, 29a Beacon Street, Boston, Mass., U. S. A., wishes to announce that it is issuing a title page and table of contents so that its publications, printed in the last two years, can easily be collected and bound by libraries and others.

THE preliminary report of the National Committee of Fifteen on Geometry Syllabus, which has been under consideration for nearly three years, has finally been published in a pamphlet of 70 pages and is ready for distribution to teachers of geometry, and all others interested. This report was prepared under the joint auspices of the American Federation of Teachers of the Mathematical and Natural Sciences and the National Education Association. It includes a historical introduction and sections on axioms and definitions, on exercises and problems, and the syllabus itself including both plane and solid geometry. It is the hope of the committee that this report may be of great service to all teachers of geometry, and to this end that it may have a wide distribution among all interested. Copies may be secured gratis upon application to the Commissioner of Education, Department of the Interior, Washington, D. C.

New Members.

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